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Application of Copulas for Cross Countries Stock Index Returns Dependency

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Abstract

Traditional correlation coefficients could not accommodate the more complexed dependency between markets. Copula modes are the alternative way to trace the dependence among markets. We use three major stock market indexes ranged from 2000 to 2020 to model the dependence between market indexes. Our sample period covers three major extreme events; therefore, it is quite suitable for copula modelling the dependence across the border. We compare the bivariate copulas to multivariate copulas in modelling the dependency. Our results show that multivariate vine copula model could choose much different pair copulas from bivariate copula. Our finding has a very important implication to portfolio manager and risk manager.

Keywords: Bivariate Copula, Vine Copula, C Vine Copula, D Vine Copula, Dependence, Correlation

1. Introduction

Traditional way to model the relation between variables is correlation which measures the dependency among variables. The crucial assumption for this measurement is normal or student-t distribution. The model based on this normal distribution did not perform well during crisis and extreme market conditions. Numerous empirical applications have proved that multivariate normal/student-t distribution is not adequate in modelling dependency. Firstly, empirical marginal distributions are often skewed and heavy-tailed. Secondly, possibilities of extreme co-movements cannot be explained by multivariate normal distribution.

Copula is the solution for the above-mentioned problems. Any multivariate distribution can serve as a copula. Copulas make it possible to model marginal distributions and the dependence structure separately. Copulas give us a greater modeling flexibility. Based on a copula, we can get many multivariate distributions by choosing different margins. The traditional representations of dependence are based on the linear correlation coefficient which is limited to multivariate elliptical distributions. Copula dependencies are free of such limitations. The copula contains all the information regarding the dependence between random variables. A copula is invariant under strictly increasing transformations. Most traditional measures of dependence are measures of pairwise dependence. Copulas measure the dependence between all random variables.

Our intention is to find out the dependency between major stock markets in the world especially when the world is more integrated since WTO that also allow openness of financial industry across the border. Instead of traditional dependence analysis, we apply more flexible method such as copulas to study the dependence among markets. More advanced copulas, like vine copulas, are also used to seek the better copula models to explain the dependency between markets.

2. Financial Applications in Copulas

2.1 Evolution of Copula Researches

Hoeffding studied properties of multivariate distributions in 1940. Sklar(1959) used the term "copula" for the first time ever. There appeared earlier financial applications (Embrechts, McNeil and Straumann, 1999, 2002). The extension of the theory of copulas to allow for conditional dependence structure was proposed by Patton (2006). Starting from 2008, copulas are widely used in finance, economics, insurance, energy, hydrology, and survival analysis.

2.2 Financial Applications of Copula

The main motivation for the use of copulas in finance comes from the growing body of empirical evidence that the dependence between many asset returns is non-normal. One important example of non-normal dependence is where two asset returns exhibit greater correlation during market downturns than during market upturns. The researches against 'copula normality' have amassed by Erb, et al. (1994), Longin and Solnik (2001), and Ang and Chen (2002), Ang and Bekaert 2002 Bae et al. 2003 among others. Since the publications of these researches, the copulas have been used for financial decision-making, in risk management, multivariate option pricing, portfolio decisions, credit risk, and studies of 'contagion' between financial markets.

The first area of application of copulas in finance was risk management. Just like 'fat tails' or excess kurtosis in the distribution of a single random variable increases the likelihood of extreme events, the appearance of non-zero tail dependence increases the likelihood of joint extreme events. In derivatives markets, non-normal dependence has key pricing and trading implications. Any option contract with two or more 'underlying' assets will generally have a price that is influenced by both the strength and the shape of the dependence between the assets. Even options with just a single underlying asset may need copulas if the risk of default by the counter-party to the contract is perceived economically significant. These types of option are called "vulnerable options".

The booming market in credit derivatives, such as credit default swaps and collateralized debt obligations, and the fact that these assets usually entail numerous underlying sources of risks has led to vast interest in copulas for credit risk applications, e.g., Li (2000) and Giesecke (2004) among others.

One of the most noticeable places where the dependence between risky assets influences financial decisions is in portfolio decisions. With quadratic utility and/or multivariate ellipticity, the optimal portfolio weights depend only upon the first two moments of the assets under consideration. However, as the joint distribution of asset returns is not elliptical or utility is not quadratic in wealth, the optimal portfolio weights will generally require a specification of all conditional distribution of returns. (Patton, 2004; Garcia and Tsafack, 2011).

The current broad topic that has attracted attention from financial economists employing copula techniques is to investigate the financial 'contagion'. Financial contagion is an occurrence which crises occur in one market cause problems in other markets beyond the expected fundamental linkages between the markets. The problem in contagion research is that a baseline level of dependence between the markets must be found before it can be claimed that the dependence escalated during a period of crisis.

From the point of portfolio and risk management perspective, understanding the dependence between asset returns is crucial. Our research intends to find out what kind of dependence between major indexes. We implement not only the bivariate copulas but also multivariate copulas to see whether or not the copulas selected would be different. The C vine and D vine copulas are implemented for the multivariate copulas.

3. Copula Models and Methodology

3.1 Traditional Dependence

Pearson linear correlation coefficient, ρ, is a simple rudimentary linear concept and it describe dependence by a single number. There are several limitations for this correlation. First of all, non-linear transformation of variables changes correlation and it needs the existence of variances. High dependence though small amount of linear correlation coefficient is possible and it does not focus on extreme dependence.

Kendall's tau, τ, normalizes difference between number of concordant and discordant pairs. Spearman's rank correlation is the correlation between ranks of data and it is not affected by extreme value. Other useful dependence measures between two variables are coefficients of low tail dependence and upper tail dependence, which are relevant to the concept of dependence in extreme values.

Ang and Chen (2002) propose to calculate the threshold correlation between two variables when two asset returns are both falling or rising more than threshold value in the meantime. On the other hand, Christoffersen et al. (2013) use the reverse threshold correlation to provide another perspective by focusing on the negative correlation between two variables when one goes up and another goes down.

3.2 Copula Models

The definition of copulas is that a *d*-dimensional copula is a multivariate distribution whose marginals are all over (0, 1). Sklar (1959) suggests a general *d*-dimensional density *h* can be expressed for some copula density *c*. This is also known as Sklar's Theorem. Copulas allow us to depict the joint distribution with two step process. First step is to estimate the appropriate marginal distributions which in not necessarily from the same family. Next is to estimate dependence structure through appropriate copula functions which could be non-linear or tail dependence. There are two commonly used copulas. One is the elliptical copula that includes Gaussian and Student-*t* copulas. The other is Archimedean copula which contains Gumbel, Clayton, Frank, mixture copula and rotated copula.

Assuming Kendall's tau =0.7 (we will discuss it next session), we simulate contours for four types of copulas in figure 1 and they are Gaussian copula, Student-*t* copula (t-copula), Clayton copula and Gumbel copula. Of course, there are more copulas than we discuss here, we just discuss the most popular ones. In figure 1, x-axis represents one dataset and y-axis is for the other one, e.g., x and y are two sets of asset returns respectively. The narrower area corresponds to higher degree of dependence. The tail areas are those in the lower left and the upper right regions.

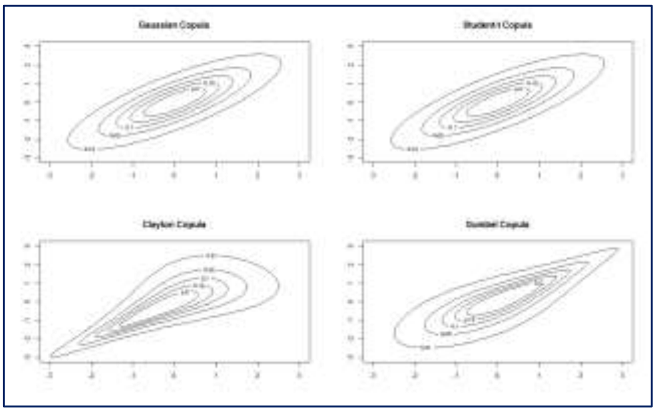

Figure 1: Four Couplas

From figure 1, we can see the one of the major characteristics for Gaussian copula is dependency in the tails goes to zero. For student-*t* copula, there is no independency in the tails, according to figure 1, but the strength of dependence in the tail increases with (1) decreasing degrees of freedom (2) increasing correlation. Regarding the Clayton copula, the graph shows us that tail dependence in the lower tail and tail independence in the upper tail. The Gumbel copula has the opposite features as the Clayton copula. That means that the Gumbel copula has tail dependence in the upper tail and tail independence in the lower tail.

For the bivariate case, a rich type of copula families is available and well-investigated (Joe 1997; Nelsen 2006). However, in arbitrary dimension, the choice of adequate families is rather limited. The high-dimension (vine) copula was initially proposed by Joe (1996) and later discussed in detail by Bedford and Cooke (2002, 2001), Kurowicka and Cooke (2006) and Aas et al. (2009). Thus vine copula allows for the specification of d(d-1)/2 bivariate copula of which the first d-1 are unconditional and the rest is conditional. The bivariate copulas involved do not have to belong to the same class. Since the decomposition is not unique, there exist many such iterative pair-copula constructions (PCC). To classify them Bedford and Cooke (2001, 2002) introduced the model called vine to help organize different PCC.

4. Data and Empirical Findings

4.1 Data

The data set contains daily log returns of three major stock index which represents three major regions in the world. They are the S&P 500 (SP500), the UK FTSE 100 index (FTSE), and the Japanese Nikkei 225 (NIKKEI). The sample period ranges from January 1, 2000 to December 31, 2020. All the holidays in three countries without trading are excluded from the data set. Therefore, there are 4883 observations in our sample. The period covers the dot.com bubble burst, the 2008 financial crisis and onset of covid-19 pandemic. The data set cover the extreme events which could be explained by copula models.

4.2 Explanatory Data Analysis

We group our data into three groups, namely SP500-FTSE, SP500-NIKKEI and FTSE-NIKKEI, to investigate the dependence conditions between these markets. The first table using traditional correlations shows that SP500 has the strongest correlation with FTSE, followed by FTSE with NIKKEI. The weakest one is between SP500 and NIKKEI. It is obvious that US market has a stronger tie to the UK market than the Japanese market due to the tradition between the two nations. The weakest correlation between US and Japan is a surprise since Japan enjoys the tremendous trade surpluses. Figure 2 also demonstrates the distributions of the three market returns and its scatter plots. From the scatter plots, it indicates that there is a stronger evidence of dependence between SP500 and FTSE than other two pair countries. The numbers in the plot are Pearson coefficients of correlation.

		SP500	FTSE	NIKKEI
SP500	Pearson	1	0.5867	0.1959
	Kendall	1	0.3672	0.1092
	Spearman	1	0.5065	0.1584
FTSE	Pearson		1	0.3724
	Kendall		1	0.2018
	Spearman		1	0.2921
NIKKEI	Pearson			1
	Kendall			1
	Spearman			1

Table 1: Correlation Matrix

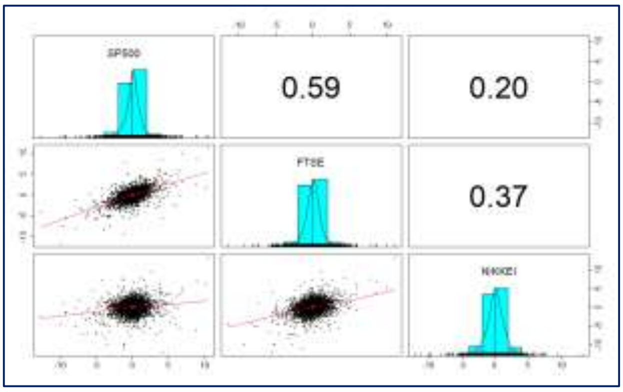

Figure 2: Scatter and Distribution Plot

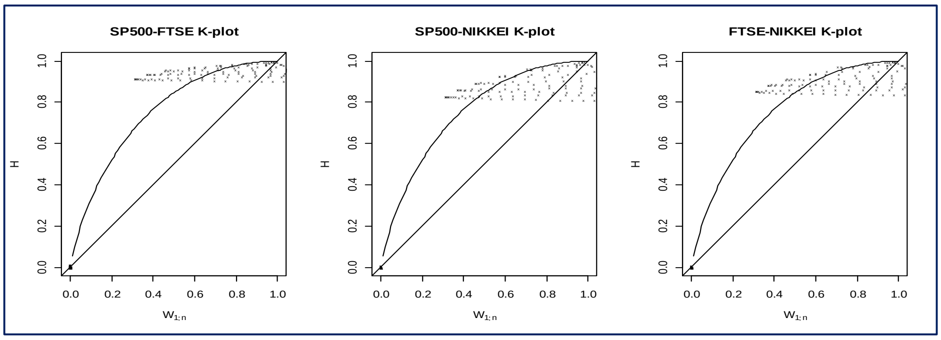

Figure 3: K-plot

Figure 3 is Kendall's plot (K-plot) for bivariate copula data of the three groups. the K-plot considers two quantities: First, the ordered values of the empirical bivariate distribution function H which is shown as the vertical axis. The second one is W in the horizontal axis. W are the expected values of the order statistics from a random sample of size N of the random variable W of the paired data. K-plots can be seen as the bivariate copula equivalent to QQ-plots. If the points of a K-plot lie approximately on the diagonal H = W, then the pair data are approximately independent of each other. Any deviation from the diagonal line points towards dependence. In case of positive dependence, the points of the K-plot should be located above the diagonal line, and vice versa for negative dependence. The larger the deviation from the diagonal, the stronger is the degree of dependency. In our case, figure 3 shows that most of time, the three groups have a strong positive dependence, with some negative dependence. The k-plot also proves that the strongest positive dependence is between SP500 and FTSE and the weakest one is between SP500 and NIKKEI. The negative dependence between all of the three markets is weak.

4.3 Empirical Findings

The bivariate copula selection is conducted first. Table 2 shows that using BIC statistics as criteria to select the appropriate copula for each group. We only demonstrate the chosen copula related statistics in table 2. Student-*t* copula is picked as the best copula for all three groups. That is not a surprised result since the patterns in the scatter plot for each group are similar in some way. The empirical taus computed from the actual data are quite close the theoretical Student-*t* copula taus, which means the Student-*t* copula fits the data quite well.

Value	SP500-FTSE	SP500-NIKKEI	FTSE-NIKKEI
Copula Chosen	Student-*t*	Student-*t*	Student-*t*
Tau	0.3668	0.1091	0.2033
Empirical Tau	0.3672	0.1092	0.2018
Log-likelihood	1139.919	180.49	356.366
AIC	-2275.838	-356.98	-708.733
BIC	-2262.851	-343.993	-695.746

Table 2 Bivariate Copula (BIC)

Furthermore, we would like to find out which family fits the data better than the other families. A goodness-of-fit score for each bivariate copula family under consideration is calculated by the Vuong and the Clarke tests which is used for bivariate copula selection. For each feasible pair of copulas families, the Vuong and the Clarke tests decide which of the two families fits the given data best and assigns a score, positive number for pro or negative number for contra a copula family. From table 3, the test scores show that student-*t* is the first choice to fit the paired dataset across three groups unanimously. The second choice would be Grumbel copula for all groups. Generally speaking, the Gumbel copula score is much lower than student-*t*, except for the SP500-FTSE paired data. This complies with the scatter plot which shows that there is some degree of strong positive dependence between SP500 and FTSE, which is one of the features of Gumbel copula, i.e., there is a tail dependence in the upper tail for Gumbel copula.

	SP500-FTSE		SP500-NIKKEI		FTSE-NIKKEI	
Copulas	Vuong	Clarke	Vuong	Clarke	Vuong	Clarke
Gaussian copula	2	0	-5	-7	1	-4
Student-*t* copula	8	8	8	8	8	8
Clayton copula	-4	-6	-1	-1	1	-2
Gumbel copula	5	3	3	3	1	2
Frank copula	-4	3	-5	-3	-4	2
Joe copula	-4	-4	-2	-2	-8	-6

Table 3 Goodness-of-Fit Test: Vuong and Clarke tests scores

The fitting process maximum log-likelihood is also carried out to select the proper copulas. According to table 4, the first choice is still student-t copula for all three groups and Grumbel copula is the second choice except the FTSE-NIKKEI group.

	SP500-FTSE	SP500-NIKKEI	FTSE-NIKKEI
Gaussian copula	871.7	75	273.6
Student-*t* copula	1140	180.5	356.4
Gumbel copula	946.3	99.03	263.6

Table 4 Fitting with Maximum Log-likelihood

We also apply high dimension vine, mainly C vine and D vine to fit our multivariate data set to see whether or not a better result will be generated. The sequential estimates of the pair-copula parameters are computed. In the meantime, we also estimate these parameters using a joint maximum log-likelihood method to see if there is a difference. The results show that two methods almost generate the same estimates. The numbers are not reported here due to limited space. Each C vine and D vine modelling are performed. To find out the better fitting vine copula model for our data set, we conduct a Vuong test by comparing both models. All the results are demonstrated in table 5.

	SP500-FTSE	SP500-NIKKEI	FTSE-NIKKEI
C Vine Copula	Student-*t* copula	Student-*t* copula	Survival Joe-Clayton copula
D Vine Copula	Student-*t* copula	Independence copula	Student-*t* copula
	statistic	*p*-value	
CDVine Vuong Test	4.82	6.71E-07	

Table 5 C and D Vine Modelling Results

When all three of the markets are considered together, instead of modelling each pair independently, the results are different. For SP500-FTSE and SP500-NIKKEI, both C Vine copula and D vine copula have selected Student-*t* copula. But it is quite different from previous bivariate copulas modelling when we select copulas. The C vine copula chose survival Joe-Clayton copula for FTSE-NIKKEI and D vine chose independence copula for SP500-

NIKKEI. Therefore, there exists a significant difference in choosing proper copulas for our data set between C vine and D vine copulas. Which one is better? The result of CDVine Vuong test shows that C vine copula is preferable to D vine copula since *p*-value is almost zero.

5. Conclusions

We are interested in how the major world stock markets are influencing one another. Since the traditional correlation could not explain the dependence between markets. The three major stock markets indexes are used to trace the dependency between the market indexes. First of all, there are indeed a relatively strong positive dependence between the markets from our data set ranged from year 2000 to 2020. The bivariate copula model is first to apply in selecting the better copula to demonstrate the dependency among countries. For these three market indexes, the student-*t* copula is the only best one to explain the dependency. But when multivariate copula models are adopted, the game has changed. Student-*t* copula is still the better choice but the dependence relationship between UK and Japanese markets could be explained by survival Joe-Clayton copula in C vine copula, but D vine chose student-*t* copula for FTSE-NIKKEI. The D vine copula even think there is an independence copula for SP500-NIKKEI but C vine chose student-*t* copula.

Our findings have demonstrated that multivariate copula models could be vital for modern portfolio management and risk management since both are related to multiple assets. There are limitations in our research. First, our study only covers three markets. Future research could investigate more markets dependency. The market opening time may play a role in determining the dependence, e.g., which market opens earlier than other markets. This issue could be addressed in the future research.

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